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Monopsony, Factor Prices, and Community Development

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MONOPSONY, FACTOR PRICES, AND COMMUNITY DEVELOPMENT

by

Lanny Arvan* and Leon N. Moses**

Abstract

We consider a simple general equilibrium model of the effects on local development and factor price determination from the entry of a large export oriented firm into a small community. The large firm has monopsony power but is not a pure monopsonist in that it must compete with a local service sector and an already situated export sector for its factor demands. Each of these sectors are taken to be comprised of many small firms which act as price takers in both the factor and product markets. Moreover, the factor supply functions are general equilibrium in nature in that the price of other factors as well as the price of local services act as determinants of factor supply. We present an extensive example where we compute the local general equilibrium and where we derive a community indirect utility function which allows us to contrast the developmental effects under the hypothesis that the large firm acts strategically in the factor markets with the hypothesis that the large firm acts as a price taker. We conclude the paper with a brief discussion of an intertemporal version of our model, where the large firm, having superior access to the (nonlocal) capital market, is motivated in part by a desire to capture arbitrage profit in the land market. The intertemporal version appears to produce a more realistic view of factor price determination than what emerges from static analysis.

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I. Introduction

This paper examines the developmental and factor price effects of entry by a large, export oriented firm into a small community. The size of the firm relative to that of the community is such that the firm's rate of output and its choice of input combinations influence the prices it must pay for local factors of production, land and labor. Because the firm has monopsonistic power it adopts a profit maximizing strategy that involves strategic quantity setting in the local factor markets.

The model used to examine these developmental and factor price effects has two main characteristics. First, it is general equilibrium in character. It deals with the impact on the output, factor usage, and price of a local goods and service sector from the large firm's entry into the community. It also investigates the effect the large firm has on the output and employment of an existing export sector that, like the local goods sector, is made up of firms that are atomistic in both their input and output markets. The second characteristic of the model derives from the fact that the large firm is not a pure monopsonist. It is dominant in local factor markets but it is not the exclusive employer, even after all of the effects of entry by the large firm have been worked out and the community is in a new long run equilibrium. In both respects the model is a departure and generalization of the traditional partial equilibrium monopsony model.

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So far as we have been able to determine, there has been little work done on extending the partial equilibrium approach in this manner. Two exceptions are Bunting (1962) and Stratton (1985). Bunting performs a cross sectional study to analyze the impact of firm concentration on wages. He finds that the anticipated negative sign of the firm size coefficient is absent. Indeed in most cases the coefficient is not statistically significant. Stratton reaches the opposite conclusion while working with a different data set. A theoretical model which considers a dominant firm in a local labor market has been advanced by Richards (1983). However, his model is flawed in that he assumes the firms which comprise the competitive fringe in the demand for labor have a perfectly elastic labor demand function. This either rules out monopsony power by the large firm or rules out operation of the competitive fringe.

In considering the impact of a large new firm on the development of a small community the necessity to take account of general equilibrium effects should be readily apparent. First, factor supply functions, in general, depend on the prices of other factors as well as on the prices of final products. Since entry by the large firm will affect these prices, the large firm's behaviour can be thought of as shifting the partial equilibrium factor supply function. As this shifting effect is not usually considered in partial equilibrium analysis, such an analysis is incomplete. Second, the factor demand function by the competitive fringe also depends on the prices of other factors and the product prices of the firms constituting the competitive fringe. The same caveat regarding curve shifting applies here as well.

Regional scientists, economists, and planners have always been very much concerned with the impacts of new industry on communities and regions. There is a massive body of literature on regional and interregional input-output and regional complex analysis that deals with such impacts and to which Isard (1951, 1953, 1965) has been a major contributor. Economic base and regional multiplier models also directly address the issue of direct and indirect employment, and sometimes even land use effects of entry into an area of a new industry. A fine summary of this literature is provided by Richardson (1985). An important critique of local multiplier studies has been given by Merrifield (1987). He points out that economic base theory suffers from an unrealistic and unpalatable factor supply assumption; factor supply functions, including that of land, are perfectly elastic. He argues for the introduction of supply side constraints into such analyses.

There is also a body of literature that deals directly with the question of the different effects of entry by a large as against entry by a number of small businesses into a community. For example Bowles (1982) in a study of Appalachia finds that large absentee firms explains much of the area's poverty. An approach to local development favoring the attraction of small business known as the "incubator approach" has received considerable attention in the literature.¹ Basquero (1987) in a study of the development of local development in Spain contrasts the growth of indigenous small firms to the entry by large firms which leave major centers of industrial agglomeration to seek the factor price advantages of peripheral areas and

¹ See Temali and Candace (1984), Small Towns (1985, 1986, 1987), Somers (1986), Brower, Pollard, and Propst (1986), and Thomas (1986) who deal with the special problems of development in small towns.

finds that much of the development of small zones in Spain resulted from growth of the former.

Regional scientists do not appear to have had a great deal of interest in modeling the large versus small firm issues in development. The regional science literature does contain an immense amount of scholarly work dealing with spatial problems of monopoly that arise on the product side. Here one must mention the recent, all inclusive and immensely impressive volume by Greenhut, Norman, and Hung (1987). However, we are unaware of any comparable treatment which deals explicitly with departures from perfect competition on the factor side.

The welfare impacts of monopsony has been given some treatment in the international trade literature. This work has been pioneered by Feenstra (1980), McCulloch and Yellen (1980), and Markusen and Robson (1980). Further work particularly interested in the development of multinationals has been done by Mendez (1984) and Markusen (1984). Because these papers adopt the Heckscher-Ohlin assumption that factor supplies are perfectly inelastic they are really incapable of examining the developmental and factor price effects of monopsony, the concern of our paper.

II. A Static Model of Entry by a Big Firm Into a Small Community

II.1 Setup of the General Framework

We consider a model of a small isolated community in which a large firm plans to locate. The community's assets are its endowments of two

factors of production, land and labor, and its endowment of the numeraire.² Prior to the entry of the large firm the community has a small industry devoted to export to the rest of the country and a sector which produces services that are consumed locally. It is assumed that both the export and local goods sector operate under constant returns to scale. The proceeds from export are spent on imports of commodities which are not locally produced. Both the export and import prices are set on national markets vis a vis which the local producers act as price takers. There is a third input which is required in production, capital. The price of capital is also set on a national market. In the short run the capital input in both the export and local goods sector is fixed and the associated fixed costs constitutes a liability for the community. In the long run each sector treats all inputs as variable.

The consumers in the community have preferences defined over consumption of final goods; the export good, the import good, and the locally produced good; as well as land and leisure consumption. Capital is not consumed. It is assumed that each consumer has preferences which can be represented by the same homothetic utility function. This assumption is made for aggregation purposes. In what follows we will consider the demand functions of the single aggregate consumer.

Let x_1 denote the export good, x_2 the import good, x_3 the locally produced good, K capital, L land, and N labor. Let the respective prices be p_1 , p_2 , p_3 , r , v , and w . p_1 , p_2 , and r are fixed from the point of view of the

² We assume that some of the community's endowment is held in a commodity which is traded with the rest of the country rather than assume that the entire endowment consists of land and labor, which are not traded, so that there is no indeterminacy in local factor prices vis a vis traded good prices. See the Cobb-Douglas example presented in the next section for details.

small community. For convenience take $p_1 = 1$, i.e., the export good will be treated as the numeraire. p_3 , v , and w are determined endogenously in the local market general equilibrium. Assuming for the moment that this general equilibrium is uniquely determined, we can view these prices as functions of the national market prices, the demand for inputs by the big firm, and in the short run the levels of the fixed capital input. In the short run the endogenous factor prices are given by

$$(1) \quad \begin{aligned} v &= v^s(p_1, p_2, r, L^d, N^d, K_1, K_3) \text{ and} \\ w &= w^s(p_1, p_2, r, L^d, N^d, K_1, K_3), \end{aligned}$$

where L^d and N^d are the demands by the big firm for land and labor, respectively and K_1 and K_3 are the levels of the fixed capital input in the export and locally produced good sectors, respectively. Similarly, in the long run the endogenous factor prices are given by

$$(2) \quad \begin{aligned} v &= v^l(p_1, p_2, r, L^d, N^d) \text{ and} \\ w &= w^l(p_1, p_2, r, L^d, N^d), \end{aligned}$$

Suppose that the product of the big firm is sold exclusively on the national market and that revenues of the big firm can be taken as a function of the joint inputs, (K^d, L^d, N^d) . Call this revenue function R . Our analysis does not depend on whether R is derived by assuming competitive or imperfectly competitive behavior in the big firm's product market. The problem which generates the big firm's demands for inputs is given by

$$(3) \quad \begin{aligned} &\text{maximize} && R(K^d, L^d, N^d) - rK^d - vL^d - wN^d. \\ &K^d, L^d, N^d \geq 0 \end{aligned}$$

Our goal is to contrast the solutions to (3) under two alternate hypotheses. First, the big firm acts strategically with respect to the local factor prices. That is the big firm is a multimarket monopsonist acting as if v and w are given by (1) in the short run or by (2) in the long run. Second, the big firm acts competitively with respect to the local factor prices taking v and w as fixed.

To pursue our goal it is highly desirable that (3) admit a unique solution. For this reason we assume that R is strictly concave. Let the big firm's short-run input expenditure function, E^S , be given by

$$(4) \quad E^S(K^d, L^d, N^d) = rK^d + vL^d + wN^d,$$

where v and w are given by (1). Similarly, let the big firm's long run input expenditure function, E^L , be given by

$$(5) \quad E^L(K^d, L^d, N^d) = rK^d + vL^d + wN^d,$$

where v and w are given by (2). It follows that strict convexity of the input expenditure function in (L^d, N^d) is a sufficient condition for uniqueness of a solution to (3). We will return to the conditions which ensure this convexity. We now turn to a more detailed development of the local, general equilibrium which generates the equations (1) and (2).

Let the local endowment of land and labor be \bar{L} and \bar{N} , respectively and let the local endowment of numeraire be \bar{m} . It is important to note that while factor endowments are fixed the big firm will perceive that there is some elasticity to factor supply functions. This follows because factors have an alternate use other than in production. That is, consumer

preferences depend on land and leisure consumption. It is in this important respect that our approach differs from the standard international trade approach. Let the local utility function be denoted by U . U is taken to satisfy the standard neoclassical properties. Then the problem which generates the consumer demands is given by

$$(6) \quad \begin{aligned} &\text{maximize} && U(x_1, x_2, x_3, L, N) \\ &x_1, x_2, x_3, L, N \geq 0 \end{aligned}$$

$$\text{Subject to: } x_1 + p_2 x_2 + p_3 x_3 + vL + wN = v\bar{L} + w\bar{N} + \pi_1 + \pi_3 + \bar{m},$$

where π_h denotes the profits earned in sector h for $h = 1$ or 3 . This yields demands of the form

$$(7) \quad \begin{aligned} x_i^* &= x_i^*(p_2, p_3, v, w, v\bar{L} + w\bar{N} + \pi_1 + \pi_3 + \bar{m}) \text{ for } i = 1, \dots, 3; \\ L_c^* &= L_c^*(p_2, p_3, v, w, v\bar{L} + w\bar{N} + \pi_1 + \pi_3 + \bar{m}); \text{ and} \\ N_c^* &= N_c^*(p_2, p_3, v, w, v\bar{L} + w\bar{N} + \pi_1 + \pi_3 + \bar{m}). \end{aligned}$$

Let the production function in sector 1 be denoted by F and the production function in sector 3 be denoted by G . Then the problem which determines the short run factor demands in sector 1 is given by

$$(8) \quad \begin{aligned} &\text{maximize} && F(K_1, L_1, N_1) - rK_1 - vL_1 - wN_1 \\ &L_1, N_1 \geq 0 \end{aligned}$$

This yields short run factor demands of the form

$$(9) \quad L_1^* = L_1^*(v, w, K_1) \quad \text{and} \quad N_1^* = N_1^*(v, w, K_1).$$

Then, $\pi_1 = F(K_1, L_1^*, N_1^*) - rK_1 - vL_1^* - wN_1^*$.

We could proceed in a similar fashion with respect to the factor demands in sector 3. Since our primary concern is with the determination of the factor prices and not with the indirect effect on the price of the locally produced good, we will instead make a simplifying assumption which greatly facilitates the analysis.

Assumption 1: Capital is not used in the production of good 3, i.e., $G(K_3, L_3, N_3) = G(0, L_3, N_3)$ for all K_3 .

Recall that we have assumed production in sector 3 is characterized by constant returns to scale. In conjunction with assumption 1 this implies that $\pi_3 = 0$, even in the short run. p_3 will be bid up or down so that there are normal economic profits in sector 3. That is,

$$(10) \quad p_3 = p_3(v, w).$$

Thus, the purpose of assumption 1 is to reduce the number of endogenous prices to be determined in equilibrium from three to two. Let C_3 denote the cost function in sector 3. Since it is assumed that there is no demand for the locally produced good from outside the community it must be that in equilibrium, $p_3 x_3^* = C_3(x_3^*)$. Thus the factor demands in sector 3 are given by

$$(11) \quad L_3^* = \sigma_3(v, w) \frac{p_3 x_3^*}{v} \text{ and } N_3^* = [1 - \sigma_3(v, w)] \frac{p_3 x_3^*}{w},$$

where $\sigma_3(v, w)$ is the cost share of land in sector 3 production and

$1 - \sigma_3(v, w)$ is the cost share of labor in sector 3 production.

Factor market clearing requires that

$$(12) \quad L_C^* + L_1^* + L_3^* = L_C^* + L_1^* + \sigma_3(v, w) \frac{p_3 x_3^*}{v} = \bar{L} - L^d \text{ and}$$

$$N_C^* + N_1^* + N_3^* = N_C^* + N_1^* + [1 - \sigma_3(v, w)] \frac{p_3 x_3^*}{w} = \bar{N} - N^d.$$

If the two equation system given in (12) is invertible, the inverse is then

— given by (1). This completes the description of the short run local general equilibrium.

In the long run sector 1 just breaks even in the equilibrium prior to the entry of the big firm. If the big firm enters at a sufficiently large scale then local factor prices are driven up enough that sector 1 must shut down. In this case sector 1 will revert to an import sector just like sector 2. Then the long run equilibrium can be solved for in a similar manner to the solution of the short run equilibrium, by assuming that production and profit in sector 1 are both zero. Alternately, the big firm may enter at small enough scale that sector 1 continues to operate, albeit at a smaller scale. In this case the local factor prices may remain unchanged, with the sector 1 local factor demands contracting to offset the local factor demands of the big firm. This case is not germane to the present analysis since the welfare effect on the community from entry by the big firm will be zero. Hence the community would have no incentive to encourage the big firm to enter in this case. It is also possible that one local factor price is bid up while the other local factor price is bid down. In general the welfare

effects on the community are ambiguous when the factor prices move in opposite directions. We briefly consider this possibility in the subsequent section.

11.2 A Cobb-Douglas Example

In this section we develop a specific example which illustrates the more general approach given above. Suppose

$$(13) \quad U(x_1, x_2, x_3, L, N) = \alpha_1 \ln x_1 + \alpha_2 \ln x_2 + \alpha_3 \ln x_3 + \alpha_L \ln L + \alpha_N \ln N,$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_L, \alpha_N > 0$ and $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_L + \alpha_N = 1$. Then the consumer demands are given by

$$(14) \quad \begin{aligned} x_i^* &= \alpha_i \frac{v\bar{L} + w\bar{N} + \pi_1 + \bar{m}}{p_i} \text{ for } i = 1, \dots, 3; \\ L_C^* &= \alpha_L \frac{v\bar{L} + w\bar{N} + \pi_1 + \bar{m}}{v} \text{ and} \\ N_C^* &= \alpha_N \frac{v\bar{L} + w\bar{N} + \pi_1 + \bar{m}}{w} \end{aligned}$$

Now suppose that the sector 1 production function is given by

$$(15) \quad F(K_1, L_1, N_1) = K_1^{\beta_K} L_1^{\beta_L} N_1^{\beta_N},$$

³ From (14) it can be seen that the purpose of the assumption that the consumer endowment contains some of the numeraire good is to avoid the possibility that in the long run, when $\pi_1 \equiv 0$, the consumer demands for land and labor depend only on the factor prices relative to each other rather than on the factor prices in respect to the numeraire. See note 1.

where $\beta_K, \beta_L, \beta_N > 0$ and $\beta_K + \beta_L + \beta_N = 1$. Then the short run factor demands in sector 1 are given by

$$(16) \quad L_1^* = \left\{ \left[\frac{\beta_L}{v} \right]^{1-\beta_N} \left[\frac{\beta_N}{w} \right]^{\beta_N} K_1^{\beta_K} \right\}^{1/\beta_K} \text{ and} \\ N_1^* = \left\{ \left[\frac{\beta_L}{v} \right]^{\beta_L} \left[\frac{\beta_N}{w} \right]^{1-\beta_L} K_1^{\beta_K} \right\}^{1/\beta_K}.$$

The short run profit in sector 1 is

$$(17) \quad \pi_1 = \beta_K K_1 \left[\frac{\beta_L}{v} \right]^{\beta_L/\beta_K} \left[\frac{\beta_N}{w} \right]^{\beta_N/\beta_K} - rK_1.$$

Finally suppose that the sector 3 production function is given by

$$(18) \quad G(L_3, N_3) = L_3^\sigma N_3^{1-\sigma}.$$

Then the factor market clearing conditions become

$$(19) \quad [\alpha_L + \alpha_3 \sigma] \frac{v\bar{L} + w\bar{N} + \beta_K K_1 \left[\frac{\beta_L}{v} \right]^{\beta_L/\beta_K} \left[\frac{\beta_N}{w} \right]^{\beta_N/\beta_K} - rK_1 + \bar{m}}{v} \\ + \left\{ \left[\frac{\beta_L}{v} \right]^{1-\beta_N} \left[\frac{\beta_N}{w} \right]^{\beta_N} K_1^{\beta_K} \right\}^{1/\beta_K} = \bar{L} - L^d$$

and

$$[\alpha_N + \alpha_3(1-\sigma)] \frac{v\bar{L} + w\bar{N} + \beta_K K_1 \left[\frac{\beta_L}{v} \right]^{\beta_L/\beta_K} \left[\frac{\beta_N}{w} \right]^{\beta_N/\beta_K} - rK_1 + \bar{m}}{w} \\ + \left\{ \left[\frac{\beta_L}{v} \right]^{\beta_L} \left[\frac{\beta_N}{w} \right]^{1-\beta_L} K_1^{\beta_K} \right\}^{1/\beta_K} = \bar{N} - N^d.$$

This rather messy system of equations is nonlinear in v and w . Nevertheless, we can still come to certain conclusions about the example.

The Long Run Case

First consider the case where $K_1 = 0$, as will certainly be true in long run equilibrium when the big firm enters at a sufficiently large scale. Then (19) simplifies to

$$(20) \quad \begin{aligned} [\alpha_L + \alpha_3 \sigma] \frac{v\bar{L} + w\bar{N} + \bar{m}}{v} &= \bar{L} - L^d \quad \text{and} \\ [\alpha_N + \alpha_3(1-\sigma)] \frac{v\bar{L} + w\bar{N} + \bar{m}}{w} &= \bar{N} - N^d. \end{aligned}$$

(20) is linear in v and w and consequently uniquely specifies the factor prices as functions of the big firm's factor demands, as long as the coefficient matrix is nonsingular. In the domain where (20) admits a sensible economic solution, we have

$$(21) \quad v = \frac{[\alpha_L + \alpha_3 \sigma](\bar{N} - N^d)\bar{m}}{D} \quad \text{and} \quad w = \frac{[\alpha_N + \alpha_3(1-\sigma)](\bar{L} - L^d)\bar{m}}{D}$$

where $D = [\bar{L} - L^d](\bar{N} - N^d) - [\bar{L} - L^d]\bar{N}[\alpha_N + \alpha_3(1-\sigma)] - \bar{L}(\bar{N} - N^d)[\alpha_L + \alpha_3 \sigma]$.

Certainly $\bar{L} - L^d, \bar{N} - N^d > 0$ is required for feasibility, i.e., the big firm does not exhaust the factor endowment in equilibrium. Then (20) admits a sensible economic solution when D is positive. This condition puts more stringent upper limits on the magnitudes of L^d and N^d . In this relevant range it follows that

$$(22) \quad \frac{\partial v}{\partial L^d}, \frac{\partial v}{\partial N^d}, \frac{\partial w}{\partial L^d}, \frac{\partial w}{\partial N^d} > 0.$$

That is, an increase in demand for either input by the big firm bids up the price of both inputs. Moreover, the factor prices are strictly convex functions of the input demands by the big firm.⁴ Since the market clearing conditions, (20), imply that the big firm's expenditure is an increasing affine transformation of the factor prices, the big firm's expenditure function is also a strictly convex function of its input demands in this case. Hence, when $K_1 = 0$ our example is amenable to the general analysis developed in the previous section.

In order to compare the equilibrium where the big firm is a monopsonist to the equilibrium where the big firm acts competitively it is instructive to construct the indirect utility of the aggregate consumer as a function of the local factor prices and then invoke the First Fundamental Welfare Theorem. This indirect utility function, Ψ , is given by

$$(23) \quad \Psi(v, w) = \ln [v\bar{L} + w\bar{N} + \bar{m}] - [\alpha_L + \alpha_3\sigma] \ln v - [\alpha_N + \alpha_3(1 - \sigma)] \ln w^5.$$

The first partials of Ψ are given by

$$(24) \quad \Psi_v = \frac{\bar{L}}{v\bar{L} + w\bar{N} + \bar{m}} - \frac{\alpha_L + \alpha_3\sigma}{v} \quad \text{and} \quad \Psi_w = \frac{\bar{N}}{v\bar{L} + w\bar{N} + \bar{m}} - \frac{\alpha_N + \alpha_3(1 - \sigma)}{w}.$$

From (20) it follows that $\Psi_v, \Psi_w > 0$ as long as $L^d, N^d > 0$. That is, as long as the big firm is actively participating in the local factor markets,

⁴ All the analytic results are given in the appendix.

⁵ Note that $\alpha_1 \ln p_1 = 0$ since $p_1 = 1$. Also note that the term $-\alpha_2 \ln p_2$ is omitted in (23). Since p_2 is constant throughout the analysis, this is just a matter of convenience.

welfare in the local community is an increasing function of the local factor prices. From this it follows that the community is indeed better off by having the big firm enter than by keeping the big firm out. Furthermore it follows that at least one factor price must be lower in the monopsony case than in the competitive case, because the First Welfare Theorem tells us that the local community is worse off when the big firm acts as a monopsonist.

Is it necessarily the case that both input demands are lower under monopsony? Note that both $\frac{\partial^2 v}{\partial L^d \partial N^d}$ and $\frac{\partial^2 w}{\partial L^d \partial N^d} > 0$. Hence $\frac{\partial^2 E}{\partial L^d \partial N^d} > 0$ as

well. Therefore, it is possible that $\frac{\partial^2 R}{\partial L^d \partial N^d} - \frac{\partial^2 E}{\partial L^d \partial N^d} < 0$ and,

apparently, one input demand might actually be larger under monopsony than under competition. $\frac{\partial^2 R}{\partial L^d \partial N^d} - \frac{\partial^2 E}{\partial L^d \partial N^d} \geq 0$ for all (L^d, N^d) is a sufficient condition for the monopsony solution to yield less demand of both inputs by the big firm and, hence, lower input prices.

What is the impact on the locally produced good sector from entry by the big firm? It follows from (18) that

$$(25) \quad p_3 = \left[\frac{v}{\sigma} \right]^\sigma \left[\frac{w}{1-\sigma} \right]^{1-\sigma}.$$

Thus the price of the locally produced good rises as the big firm bids up the local factor prices. It then follows, by substituting (25) into (14), that a proportionate increase in both factor prices actually lowers the equilibrium demand for good 3. Hence there is a tendency for sector 3 to contract as the big firm enters. However, if the cost shares in sector 3 are

very uneven and if the big firm disproportionately bids up the price of the factor which has the small cost share in sector 3, then it is possible for sector 3 to expand upon entry by the big firm.

We now consider the case where $K_1 > 0$ after entry by the big firm. That is, entry by the big firm occurs at a small enough scale that continued operation by the old export sector is possible. Note that the average cost function in sector 1 is given by

$$(26) \quad \frac{C_1(x_1)}{x_1} = \left[\frac{r}{\beta_K} \right]^{\beta_K} \left[\frac{v}{\beta_L} \right]^{\beta_L} \left[\frac{w}{\beta_N} \right]^{\beta_N}.$$

Since good 1 is the numeraire and since the profits in sector 1 are zero in the long run, as long as sector 1 is in operation the local factor prices are constrained to satisfy

$$(27) \quad \beta_L \ln v + \beta_N \ln w = H,$$

where $H = \beta_K \ln \frac{\beta_K}{r} + \beta_L \ln \beta_L + \beta_N \ln \beta_N$. From (27) it is evident that if one local factor price is bid up the other local factor price must be bid down. The change in w given a unit increase in v so that (27) remains satisfied is given by

$$(28) \quad \frac{dw}{dv} = - \frac{\beta_L}{\beta_N} \frac{w}{v}.$$

Then using (24), the welfare effect of such a price change is

$$(29) \quad \Psi_v + \Psi_w \frac{dw}{dv} = \frac{\bar{L}}{v\bar{L}+w\bar{N}+\bar{m}} - \frac{\alpha_L + \alpha_3 \sigma}{v} - \frac{\beta_L w}{\beta_N v} \left(\frac{\bar{N}}{v\bar{L}+w\bar{N}+\bar{m}} - \frac{\alpha_N + \alpha_3(1-\sigma)}{w} \right).$$

(29) is positive if and only if

$$(30) \quad \frac{v\bar{L} - [\alpha_L + \alpha_3 \sigma][v\bar{L} + w\bar{N} + \bar{m}]}{\beta_L} > \frac{w\bar{N} - [\alpha_N + \alpha_3(1-\sigma)][v\bar{L} + w\bar{N} + \bar{m}]}{\beta_N}.$$

The numerator of the left hand side of (30) is the value of the excess supply of land for the community, inclusive of the indirect demand for land in local goods production but exclusive of the demand for land by either the exporting sector or the big firm. The numerator of the right hand side of (30) is similarly the value of the excess supply of labor. Thus the community benefits from an increase in the price of land and a concomitant decrease in the price of labor if and only if the ratio of the value of excess land to labor supply exceeds the ratio of the land to labor cost shares in sector 1 production.

If the local export sector continues to operate after entry by the big firm the community will only invite the big firm in if (30) is satisfied and the community expects the big firm to bid up the price of land or if (30) is satisfied in reverse and the community expects the big firm to bid up the price of labor.

Since $\pi_1 = 0$ in the long run, (19) can now be written as

$$(31) \quad [\alpha_L + \alpha_3 \sigma] \frac{v\bar{L} + w\bar{N} + \bar{m}}{v} + \beta_L x_1 = \bar{L} - L^d \text{ and}$$

$$[\alpha_N + \alpha_3(1-\sigma)] \frac{v\bar{L} + w\bar{N} + \bar{m}}{w} + \beta_N x_1 = \bar{N} - N^d,$$

where x_1 is the output of the local export sector. Observe that if the big firm expands its demands of both land and labor so that $\frac{dN^d}{dL^d} = \frac{\beta_N}{\beta_L}$ and if in the process the local export sector contracts its demands so that $\frac{dx_1^1}{dL^d} = \frac{-1}{\beta_L}$, then neither local factor price is affected. That is, as long as the big firm increases its local factor demands in the correct proportion, then the big firm views the supply of local factors as perfectly elastic, regardless of whether the big firm acts strategically or as a price taker in the local factor markets. Thus when the local export sector continues to operate the following first order condition is necessary at the big firm's optimum.

$$(32) \quad \beta_L (R_L - v) + \beta_N (R_N - w) = 0.$$

Of course when the big firm acts competitively in the local factor markets then both $R_L = v$ and $R_N = w$. When the big firm acts strategically this need not be the case.

Note that the expenditure function E^1 need not be convex in (L^d, N^d) as long as the local export sector continues to operate. Thus we cannot rule out multiple solutions to the big firm's problem in this case. However, it is not hard to show that for each output level of the local export sector, x_1 , there is a unique (v, w) pair which satisfies (27), (31), and is profit maximizing for the big firm.

Short Run Analysis

Obviously we cannot obtain an explicit analytic solution for the factor prices given by (19) when $K_1 > 0$. Nevertheless in the region where (19) admits an economic solution this solution is unique.⁶ Moreover it is still the case that.

$$(33) \quad \frac{\partial v}{\partial L^d}, \frac{\partial w}{\partial N^d} > 0.$$

However, in the short run it is possible that $\frac{\partial v}{\partial N^d}, \frac{\partial w}{\partial L^d} < 0$. In the long run

case bidding up the price of one input raises the local demand for the other input via income effects. In the short run this effect is still present but the demand for the other input in the export producing sector is reduced. The export producing sector is squeezed by the increase in factor demand from the big firm and responds by a decrease in its own factor demands. Hence the overall effect on the local demand for the other input is ambiguous. Convexity of the factor prices in the big firm's factor demands is still a sufficient condition for convexity of the the big firm's expenditure function. Finally, the local community's welfare is increasing in the local factor prices given that the big firm is taking a positive position in the local factor markets. Thus the same caveats concerning the monopsony solution in the long run case are also applicable to the short run analysis.

⁶ Apart from restrictions on the size of the big firm's factor demands it is necessary that the level of the fixed capital input in the export producing sector not be too large. By limiting the size of the fixed cost in this sector positive net wealth of the community can be ensured.

III. An Intertemporal Approach

While the static model developed in the previous section is an improvement on the partial equilibrium approach to monopsony, it appears somewhat at odds with some stylized facts concerning the way local communities in the United States appear to woo large firms. Casual empiricism suggests that small communities try to attract large firms because they expect a substantial boon to local employment and because they also anticipate a concomitant increase in local land values. While the static long run model predicts increases in both factor prices relative to the preentry equilibrium, the strategic behavior by the large firm seemingly mutes these effects. Yet recently it appears that big firms often pay the national industrial wage upon entry into a small community. If this is correct then this fact would appear to be at odds with the predictions of our static model.

Recent theories concerning "efficiency wages" may explain why the big firm finds it advantageous to pay such high wages when it possesses monopsony power.⁷ But if we are to rely on efficiency wage theory to explain the high wages offered by the big firm, and note that the local reservation wage is likely to be substantially lower than the urban reservation wage even after entry by the big firm, then there is only the land market in which the big firm can exercise its strategic power. But as we have already noted, there is the anticipation that local land values will rise substantially as well. We believe that there is something else going

⁷ For a good survey of the efficiency wage literature see the volume edited by Akerlof and Yellen (1986).

here. Below we provide a possible explanation consistent with these stylized facts.

Our point of departure is in the assumption that the big firm has better access to credit markets than does the local community. This assumption, together with the knowledge that local factor prices will rise as a consequence of its entry, provides the big firm with an incentive to act as an arbitrageur in the land market. That is, prior to the time that the big firm has established its operation in the local community, the big firm buys up land in excess of what is needed for its operation. Once its operation has been fully established and local factor prices have been bid up as a consequence, it sells back the excess land to the community and thereby earns a handsome profit.

If this channel for exercising strategic power is open to the big firm then the big firm should base its location choice, in part, on the community's access to credit, preferring more isolated unknown locations. Moreover, if the land market is vertically integrated with the local goods and services market then this form of arbitrage will take the form of the big firm becoming a significant player in local industry. In other words, this theory is consistent with the notion of the development of a company town.

To get an idea of how this arbitrage works consider an infinite horizon, discrete time model. Suppose that in each period the model is similar to the static model developed in section II.1. That is, the utility function, U , is now to be interpreted as the per period utility function and

time preference for the community is determined by the discount rate δ .⁸ In the period prior to its operation the big firm buys up land in the community but does not participate in the local labor market. Assume that in all subsequent periods the big firm is in operation and a stationary equilibrium is established.⁹ The per period equilibrium of all periods but the interim period, in which the big firm buys up land, is described by the static model we have already developed with the exception that the community's endowment of land is now net of what it sold to the big firm in the interim period and the big firm now may be a supplier rather than a demander of land.

Let v_S be the stationary equilibrium rental price of land and let i_S be the long run interest rate in the community. Then the stationary equilibrium land price, p_S , is given by

$$(34) \quad p_S = \frac{1+i_S}{i_S} v_S.$$

Similarly, the interim equilibrium price of land in the community in the period when the big firm buys up land, p_I , is given by

$$(35) \quad p_I = v_I + \frac{p_S}{1+i_I} = v_I + \frac{1+i_S}{1+i_I} \cdot \frac{v_S}{i_S},$$

⁸ This intertemporal additive separability of preferences implies zero intertemporal cross price effects in demand, which greatly facilitates the analysis.

⁹ This requires that the long run interest rate which the community faces equals the community's rate of time preference.

where v_l is the rental price of land and i_l is the short run interest rate in the interim period.¹⁰ If $v_S > v_l$ because the big firm's demand for labor is sufficient to bid up the price of land in the long run, more than offsetting the arbitrage in the land market by the big firm, then there is a capital gain in land as long as $i_l \geq i_S$. We take this as our starting point.

Assume that consumers in the local community have rational point expectations as to the long run equilibrium prices once the big firm is in operation. The question which concerns us here is how does strategic play by the big firm once it is in operation impact on the price of land in the interim period? In particular, for a given level of land purchased by the big firm in the interim period, how does an increase in the big firm's long run demand for labor or a decrease in its supply of land affect p_l ? It turns out that p_l is likely to be inelastic with respect to such strategic moves by the big firm and to be more inelastic the smaller is the community's elasticity of credit.¹¹

Consider the extreme case where all loans in the community are self-financing, so that in the absence of entry by the big firm the community must maintain balanced trade with the rest of the world. We will show that in this extreme case such strategic play by the big firm has no effect at all on v_l . Let L_l^d denote the amount of land bought up by the big firm in the interim period. Then the stationary equilibrium with the big firm in operation is determined by the same set of equations that we have

¹⁰ Since we are abstracting from any considerations of risk, the short run interest rate will depart from the long run interest rate only if the community's supply of credit is imperfectly elastic.

¹¹ Of course, when the supply of credit to the community is perfectly elastic the anticipated increase of the land rental price will bid up p_l substantially. In this case, p_S will exceed p_l only by the difference between v_S and v_l .

already examined in the static model, with the exception that the

community's endowment of land in this stationary equilibrium is given by $\bar{L} - L_1^d$ and the big firm may very well be a net supplier of land.¹² Since

there are no intertemporal cross price effects in demand, the local community's demand for land and labor services in the interim period is a function of the exogenously given prices of the imported and exported goods, the interim period rental prices, v_1 and w_1 , as well as the lifetime

wealth of the community, $p_1 \bar{L} + w_1 \bar{N} + \bar{m} + \frac{1+i_S}{1+i_1} \frac{w_S \bar{N} + \bar{m}}{i_S} =$

$v_1 \bar{L} + w_1 \bar{N} + \bar{m} + \frac{1+i_S}{1+i_1} \frac{v_S \bar{L} + w_S \bar{N} + \bar{m}}{i_S}$. Moreover, given our restriction on

preferences, there is a unique pair of factor prices and level of community wealth which gives rise to a given land and labor demand pair. Now

suppose in the interim period consumers in the community rationally anticipate a long run increase in the big firm's demand for labor but witness no change in the amount of land the big firm buys up in the interim period. This increase in labor demand will bid up w_S and v_S . However, from the above v_1 and w_1 should remain unchanged. Thus, in this extreme case i_1

adjusts so that the community perceives no increase in its lifetime wealth.

In other words, in this extreme case rationally anticipated changes in the big firm's stationary equilibrium behavior will have no effect on the interim rental prices. Such changes will manifest themselves only through the changes in the short run interest rate. Indeed, if v_S and $1+i_1$ increase in the same proportion then there is no change in p_1 whatsoever.

¹² This would be the case if the amount of land bought up in period 1 exceeds the big firm's use of land in production.

The point here is that in deciding on its optimal strategic play once it has obtained full operation the big firm can more or less ignore the impact this play has on its preentry price of land. Since in the long run equilibrium the big firm is operating as a monopsonist in the labor market but as a monopolist in the land market it is quite conceivable that the long run wage is competitive or even supercompetitive and that land prices are quite high.

IV. Conclusion

In this paper we have been concerned with developing a general equilibrium framework for analyzing the impacts of entry by a large firm into a small community which is consistent with the notion that communities bid to attract such firms because they generate substantial income locally. Far from feeling that we have offered a complete analysis, we hope that our paper will stimulate further work in the area. Below we present some points in which the current analysis is deficient and which might prove fruitful for study.

First, this paper completely ignores the effects of labor migration into the community from adjacent areas. Such effects are obviously of great interest to regional scientists. Moreover, reconciling such migration with the fact that big firms pay high wages is all the more perplexing because it would seem that the such labor migration would make the big firm's labor supply more elastic than is suggested by the analysis in this paper.

Second, other channels apart from different access to credit markets should be explored in regard to the big firm's ability to act as an arbitrageur in the land market. In this paper we have ignored the fact that prior to entry by the big firm, the local community is typically in competition with other communities for the plant that the big firm will set up. Such competition may lend an air of uncertainty which is absent in our model. Is it conceivable that the big firm can utilize this uncertainty of its location choice to aid it in land speculation?

Finally, we have not addressed the issue of community size. In other words, in the static framework a sensitivity analysis on how the big firm's profits vary with the basic parameters of our model would be of great interest.

Furthermore, while we have argued that the big firm would like to enter a community which has limited access to credit markets in the intertemporal model, obviously the big firm would prefer that its labor supply is plentiful. An examination of how this tradeoff affects the big firm's location choice would also be highly desirable.

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Appendix on Cobb-Douglas Example

Long Run Analysis

Since $D = [\bar{L} - L^d][\bar{N} - N^d] - [\bar{L} - L^d]\bar{N}[\alpha_N + \alpha_3(1-\sigma)] - \bar{L}[\bar{N} - N^d][\alpha_L + \alpha_3\sigma]$ and since $\bar{L} - L^d, \bar{N} - N^d > 0$, it follows that

$$\bar{N} - N^d - \bar{N}[\alpha_N + \alpha_3(1-\sigma)] > 0 \quad \text{and} \quad \bar{L} - L^d - \bar{L}[\alpha_L + \alpha_3\sigma] > 0$$

are necessary conditions for $D > 0$. Then the precise comparative static results are

$$\frac{\partial v}{\partial L^d} = \frac{v}{D} \{ \bar{N} - N^d - \bar{N}[\alpha_N + \alpha_3(1-\sigma)] \} > 0 \quad \text{and}$$

$$\frac{\partial v}{\partial N^d} = \frac{v}{D} \left\{ \frac{[\bar{L} - L^d]\bar{N}[\alpha_N + \alpha_3(1-\sigma)]}{\bar{N} - N^d} \right\} > 0.$$

Next we can obtain the second partials as follows

$$\frac{\partial^2 v}{\partial L^d{}^2} = 2 \frac{v}{D^2} \{ \bar{N} - N^d - \bar{N}[\alpha_N + \alpha_3(1-\sigma)] \}^2 > 0,$$

$$\begin{aligned} \frac{\partial^2 v}{\partial L^d \partial N^d} = & - \frac{v}{D} \left\{ \frac{\bar{N}[\alpha_N + \alpha_3(1-\sigma)]}{\bar{N} - N^d} \right\} + \\ & 2 \frac{v}{D^2} \{ \bar{N} - N^d - \bar{N}[\alpha_N + \alpha_3(1-\sigma)] \} \left\{ \frac{[\bar{L} - L^d]\bar{N}[\alpha_N + \alpha_3(1-\sigma)]}{\bar{N} - N^d} \right\} > 0 \end{aligned}$$

because $\frac{\{\bar{N}-N^d-\bar{N}[\alpha_N+\alpha_3(1-\sigma)]\}[\bar{L}-L^d]}{D} > 1$, and

$$\frac{\partial^2 v}{\partial N^d{}^2} = \frac{v}{D} \left\{ \frac{[\bar{L}-L^d]\bar{N}[\alpha_N+\alpha_3(1-\sigma)]}{[\bar{N}-N^d]^2} \right\} + 2 \frac{v}{D^2} \left\{ \frac{[\bar{L}-L^d]\bar{N}[\alpha_N+\alpha_3(1-\sigma)]}{\bar{N}-N^d} \right\}^2 > 0.$$

Moreover, $\frac{\partial^2 v}{\partial L^d{}^2} \frac{\partial^2 v}{\partial N^d{}^2} - \left[\frac{\partial^2 v}{\partial L^d \partial N^d} \right]^2 =$

$$\frac{v^2}{D^3} \{\bar{N}-N^d-\bar{N}[\alpha_N+\alpha_3(1-\sigma)]\}^2 \left\{ \frac{[\bar{L}-L^d]\bar{N}[\alpha_N+\alpha_3(1-\sigma)]}{[\bar{N}-N^d]^2} \right\} -$$

$$\frac{v^2}{D^2} \left\{ \frac{\bar{N}[\alpha_N+\alpha_3(1-\sigma)]}{\bar{N}-N^d} \right\}^2 +$$

$$4 \frac{v^2}{D^3} \{\bar{N}-N^d-\bar{N}[\alpha_N+\alpha_3(1-\sigma)]\} \left\{ \frac{[\bar{L}-L^d]\bar{N}[\alpha_N+\alpha_3(1-\sigma)]}{\bar{N}-N^d} \right\} \left\{ \frac{\bar{N}[\alpha_N+\alpha_3(1-\sigma)]}{\bar{N}-N^d} \right\} =$$

$$3 \frac{v^2}{D^3} \{\bar{N}-N^d-\bar{N}[\alpha_N+\alpha_3(1-\sigma)]\} \left\{ \frac{[\bar{L}-L^d]\bar{N}[\alpha_N+\alpha_3(1-\sigma)]}{\bar{N}-N^d} \right\} \left\{ \frac{\bar{N}[\alpha_N+\alpha_3(1-\sigma)]}{\bar{N}-N^d} \right\} +$$

$$\frac{v^2}{D^2} \left\{ \frac{[\bar{L}-L^d]\bar{N}[\alpha_N+\alpha_3(1-\sigma)]}{\bar{N}-N^d} \right\} \times$$

$$\left\{ \frac{\{\bar{N}-N^d-\bar{N}[\alpha_N+\alpha_3(1-\sigma)]\}[\bar{L}-L^d]}{D} - \left\{ \frac{\bar{N}[\alpha_N+\alpha_3(1-\sigma)]}{\bar{N}-N^d} \right\} \right\} > 0$$

where the last inequality follows because $\frac{\{\bar{N}-N^d-\bar{N}[\alpha_N+\alpha_3(1-\sigma)]\}[\bar{L}-L^d]}{D} > 1$

and $\frac{\bar{N}[\alpha_N + \alpha_3(1-\sigma)]}{\bar{N} - N^d} < 1$. This demonstrates that v is strictly convex and increasing in the big firm's factor demands when these factor demands are in the relevant range. Since the equilibrium is symmetric the same results also apply to w .

Short Run Analysis

The market clearing conditions (19) can be rewritten as

$$[\alpha_L + \alpha_3\sigma] \frac{v\bar{L} + w\bar{N} + \pi_1 + \bar{m}}{v} + L_1^* = \bar{L} - L^d \text{ and}$$

$$[\alpha_N + \alpha_3(1-\sigma)] \frac{v\bar{L} + w\bar{N} + \pi_1 + \bar{m}}{w} + N_1^* = \bar{N} - N^d.$$

Existence of a solution follows by standard arguments since the demands are well behaved. For uniqueness of a solution consider the Jacobian of this system

$$\begin{bmatrix} \frac{-[\alpha_L + \alpha_3\sigma][vL_1^* + w\bar{N} + \pi_1 + \bar{m}]}{v^2} + \frac{\partial L_1^*}{\partial v} & \frac{[\alpha_L + \alpha_3\sigma][\bar{N} - N_1^*]}{v} + \frac{\partial L_1^*}{\partial w} \\ \frac{[\alpha_N + \alpha_3(1-\sigma)][\bar{L} - L_1^*]}{w} + \frac{\partial N_1^*}{\partial v} & \frac{-[\alpha_N + \alpha_3(1-\sigma)][v\bar{L} + wN_1^* + \pi_1 + \bar{m}]}{w^2} + \frac{\partial N_1^*}{\partial w} \end{bmatrix}$$

This Jacobian is nonsingular. Indeed the matrix is negative definite since it is the sum of

$$\begin{bmatrix} \frac{-[\alpha_L + \alpha_3 \sigma][vL_1^* + w\bar{N} + \pi_1 + \bar{m}]}{v^2} & \frac{[\alpha_L + \alpha_3 \sigma](\bar{N} - N_1^*)}{v} \\ \frac{[\alpha_N + \alpha_3(1 - \sigma)](\bar{L} - L_1^*)}{w} & \frac{-[\alpha_N + \alpha_3(1 - \sigma)][v\bar{L} + wN_1^* + \pi_1 + \bar{m}]}{w^2} \end{bmatrix}$$

and

$$\begin{bmatrix} \frac{\partial L_1^*}{\partial v} & \frac{\partial L_1^*}{\partial w} \\ \frac{\partial N_1^*}{\partial v} & \frac{\partial N_1^*}{\partial w} \end{bmatrix}$$

both of which are negative definite. This implies that each solution is isolated. It also implies the comparative statics results suggested in the text in (25).

Next consider the function H given by

$$H(v, w) = \{v[\bar{L} - L_1^d] - [\alpha_L + \alpha_3 \sigma][v\bar{L} + w\bar{N} + \pi_1 + \bar{m}] - vL_1^*\}^2 + \\ \{w[\bar{N} - N_1^d] - [\alpha_N + \alpha_3(1 - \sigma)][v\bar{L} + w\bar{N} + \pi_1 + \bar{m}] - wN_1^*\}^2.$$

Note that from (16) and (17) it follows that π_1 , vL_1^* , and wN_1^* are all strictly concave in (v, w) . Hence $H(v, w)$ is convex in (v, w) . Moreover for w fixed H explodes as v approaches either 0 or ∞ . Similarly for v fixed H explodes as w approaches either 0 or ∞ . It follows that H attains its minimum over the first quadrant at some strictly positive (v, w) . Moreover, the set of such minimizers must be convex. In fact any solution to (19) must

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be such a minimizer of H because H is obviously nonnegative and is zero at a solution to (19). It follows that the solution to (19) is unique.

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